The University of Chicago

ELECTRIC FIELDS PRODUCED BY COSMIC RAYS

A DISSERTATION SUBMITTED TO THE FACULTY OF THE DIVISION OF THE PHYSICAL SCIENCES IN CANDIDACY FOR THE DEGREE OF DOCTOR OF PHILOSOPHY

DEPARTMENT OF PHYSICS

1941

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FOSTER EVANS

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Reprinted from
THE PHYSICAL REVIEW, Vol. 59, January 1, 1941

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Reprinted from THE PHYSICAL REVIEW, Vol. 59, No. 1, 1–7, January 1, 1941
Printed in U. S. A.

Electric Fields Produced by Cosmic Rays

FOSTER EVANS
Ryerson Physical Laboratory, The University of Chicago, Chicago, Illinois
(Received August 23, 1940)

With recognition of the existence in interstellar space of a finite density of matter, the following four assumptions are made regarding the conditions under which electric currents flow in interstellar space: (1) The current is related to the electric field producing it by Ohm's law; (2) the interchange of electric charge between a star and its surroundings has reached a steady state; (3) the conditions around a star are spherically symmetric, which implies that the star has no magnetic moment; (4) the forces due to gravity, radiation pressure, and diffusion can be neglected simultaneously. The validity of these assumptions is discussed. The electric field E in the neighborhood of a single star is given by the equation, $A = -\sigma r^2 E$, where A is proportional to the rate at which the star intercepts, or emits, cosmic-ray particles all of the same charge, σ is the electrical conductivity in interstellar space, and r is the distance of the point under consideration from the center of the star. It is found that both charge density and current density vanish in interstellar space. Numerical calculations indicate that the star will acquire a very small potential when it only intercepts cosmic rays. For a typical star this is of the order of 10^{-12} volt. In the case of emission an upper limit to the potential is found to be approximately one volt. The limitations of the applicability of the present analysis for the case of emission are discussed. It is concluded that the existence in interstellar space of cosmic rays as charged particles predominantly of one sign will result in the production of negligible electric fields. Calculations are made to determine the dimensions of the ion sheath that will be produced around a star by the ions in interstellar space.

INTRODUCTION AND STATEMENT OF ASSUMPTIONS

IN theoretical investigations concerning the source of primary cosmic-ray particles and their subsequent transmission through interstellar space, the problem of the possible electric fields produced is an important one. Experimental evidence indicates that both positively and negatively charged primaries enter the earth's atmosphere, but whether there is an appreciable excess of particles of one sign in space is not known. It is true that the latitude effect indicates that positive primaries produce more effect at the earth's surface than do negative primaries. This might indicate an excess of positives in space, or merely a difference in penetrating power. It has been the purpose of this investigation to consider the electrical fields that would result from a net charge carried by the primary rays. Some years ago Swann[1] showed that if cosmic rays were present in empty space as charged particles predominantly of one sign, the resultant space charge would produce enormous potential differences between relatively near points in space. His calculations indicated further that: "Any attempt to picture particles of one sign as emanating from the stars or other heavenly bodies would result in a condition in which the particles would become bound to their respective stars as a sort of space charge unless both signs of charged particles were emitted." Strictly, however, it is not correct to consider

[1] W. F. G. Swann, Phys. Rev. 44, 124 (1933).

interstellar space as being empty. Astronomical observations show evidence for the existence of a finite density of electrons and ions. Swann pointed out that the presence of such interstellar charges would neutralize the space charge caused by cosmic rays. Working upon this hypothesis Alfvén[2] has made some calculations from which he concluded that the magnetic field produced by the electric current caused by cosmic rays would have a limiting effect upon the amount of such current.

In order to consider the hypothesis of space charge neutralization in more detail the following assumptions—the validity of which will be discussed later—are made. (1) Ohm's law is valid for electric currents flowing in the very rare gas which fills interstellar space. (2) The interchange of electric charge between a star and its surroundings has reached a steady state. (3) The conditions around the star are spherically symmetric. As implied in this assumption, the effect of stellar magnetic moments has been neglected. (4) The forces due to gravity, radiation pressure, and diffusion can be neglected simultaneously for small electric fields.

Derivation of Equations

Consider a single star immersed in the gas which fills interstellar space, and let the constant $4\pi A$ be the total amount of electric charge intercepted, or emitted, by the star per second in the form of cosmic rays. There will also be a current flow to the star resulting primarily from the motion of interstellar electrons. Throughout this discussion the expression "interstellar electrons" will refer to the slow electrons that occupy interstellar space and are not to be confused with high energy electrons which may make up a part of the cosmic radiation. The current due to the interstellar electrons is given, according to the first assumption, by the expression

$$i_{\text{elect}} = 4\pi r^2 \sigma E, \tag{1}$$

where σ is the electrical conductivity of the interstellar gas and E is the electric field in which the electrons are moving at a distance r from the center of the star. This current plus that caused by the cosmic rays, given by

$$i_{\text{c.r.}} = 4\pi A, \tag{2}$$

represents the total amount of current flowing across the boundary of a sphere of radius r drawn with its center at the center of the star. Both currents are independent of r, according to the second assumption. Furthermore, the algebraic sum of these two currents is zero since electricity is not created or destroyed in the star. Therefore, by adding the right-hand sides of Eqs. (1) and (2) and equating to zero, we obtain, after solving for the electric field E,

$$E = -A/\sigma r^2. \tag{3}$$

If ρ, ρ_+ and $\rho_{\text{c.r.}}$ are, respectively, the charge densities caused by electrons, positive ions, and cosmic rays, it is possible to write

$$\text{div } E = 4\pi(\rho + \rho_+ + \rho_{\text{c.r.}})$$

and from Eq. (3) it is seen that

$$\rho + \rho_+ + \rho_{\text{c.r.}} = 0, \tag{4}$$

which is in agreement with the hypothesis of space charge neutralization. In other words, E is caused entirely by a charge $Q = A/\sigma$ on the star. Further, since a condition of spherical symmetry surrounds the star it follows that the current density is equal to zero at all points and there is no magnetic field caused by cosmic rays.

Alfvén[3] states that a process similar to that just described, that is an isotropic emission of equal numbers of positive and negative particles from the surface of a sphere, is unstable if the medium around the sphere is conducting. This statement, however, seems untenable in view of the fact that spherical electrodes immersed in the positive column of a discharge tube have been observed to be surrounded by stable, spherical ion sheaths through the surfaces of which passed equal numbers of positive and negative particles.

The conductivity σ may be written as the product of the charge density ρ caused by the interstellar electrons, and the mobility κ. From kinetic theory the mobility is given by

$$\kappa = e\lambda/mu,$$

[2] A. Alfvén, Phys. Rev. 55, 425 (1939).

[3] Reference 2, p. 427.

where e is the charge, λ the mean free path, m the mass and u the average velocity of the electron. Again from kinetic theory

$$u=(3kT/m)^{\frac{1}{2}}, \quad \text{and} \quad \lambda=1/\pi s^2 n,$$

where k is Boltzmann's constant, T is the translation temperature of interstellar electrons, s is the sum of the radius of the electron and the average radius of the particles in interstellar space and n is the total density of such particles. Substitute these formulae for u and λ into the formula for κ and there results

$$\kappa=[e/\pi s^2(3mkT)^{\frac{1}{2}}](1/n).$$

Now let β be the fraction of the total density of the interstellar gas that consists of electrons, and the charge density of electrons becomes $\rho=e\beta n$. Then, since $\sigma=\rho\kappa$ it is found upon substitution into Eq. (3) that

$$E=-\pi s^2(3mkT)^{\frac{1}{2}}A/e^2\beta r^2, \qquad (5)$$

which indicates that the electric field E is independent of the density n. Upon integration of Eq. (5) it is seen that the potential produced on a star of radius a as a result of interception or emission of cosmic rays is given by

$$V=-\pi s^2(3mkT)^{\frac{1}{2}}A/e^2\beta a. \qquad (6)$$

Interstellar Electrons

At this point it seems advisable to digress slightly in order to give a brief outline of the present ideas concerning electrons in interstellar space. In a recent publication by Dunham[4] it is calculated that in interstellar space between the earth and χ^2 Orionis the density of electrons is approximately equal to $10/\text{cm}^3$. He suggests, however, that it is probable that this value should be reduced by a factor of ten or more. Dunham determines a value for the density of hydrogen which is approximately equal to that of electrons, while the total density of sodium, potassium, calcium and titanium is less than 1.4×10^{-4}. This indicates that the fraction β used in the above formulae is approximately equal to $\frac{1}{2}$. The translation temperature of interstellar gas which is usually adopted in astrophysical calculations is of the order of[5, 6] 10,000°K.

[4] T. Dunham, Jr., Proc. Am. Phil. Soc. **81**, 277 (1939).
[5] A. Corlin, Zeits. f. Astrophysik **18**, 1, 11 (1939).
[6] O. Struve, Proc. Nat. Acad. Sci. **25**, 37 (1939).

Discussion of Assumptions

A necessary condition for the application of Ohm's law is that the velocity gained by an interstellar electron over the mean free path shall be small compared with the average velocity of temperature agitation. The limit within which this requirement is fulfilled is discussed below. Also, in order that Ohm's law be obeyed for the conduction of electric currents through the interstellar gas, it is necessary that an electron make enough collisions with atomic particles in interstellar space to reach a terminal velocity before it collides with a star. In other words, the interstellar electrons must make many more collisions with atomic particles than they do with stars. In order to determine whether or not this is so, the approximate mean free paths L_1, for electrons colliding with atomic particles, and L_2 for electrons colliding with stars were calculated from the formula for mean free path given above. In order to obtain L_1, let the density N_1 of interstellar particles be equal to twice the density of electrons. In view of the discussion in the previous section this latter density will be taken as one electron per cubic centimeter. Let s_1, the sum of the radii of the electron and the particles of atomic size which consist principally of hydrogen atoms and ions, be one angstrom, then

$$L_1=1/\pi s_1^2 N_1=1.68\times10^{-3} \text{ light yr.}$$

Let the density N_2 of the stars in our galaxy be 0.1 per cubic parsec,[7] and let the average stellar radius (see below) be the radius of the sun, i.e., $s_2=6.95\times10^{10}$ cm, then

$$L_2=1/\pi s_2^2 N_2=2.05\times10^{16} \text{ light yr.}$$

By comparing L_1 and L_2 it is seen that the probability of an electron colliding with a star is much smaller than that of its colliding with an interstellar particle of atomic size. Suppose that instead of there being one or more electrons per cubic centimeter in interstellar space, the average electron density is as low as 10^{-3} per cm^3. Then L_1 would be of the order of magnitude of the average distance between stars in the galaxy. Although in these circumstances an electron might travel such a distance before making a

[7] J. H. Jeans, *Astronomy and Cosmogony*, second edition (Cambridge, 1929), p. 15.

collision, there still would be very little likelihood of its colliding with a star because of the small cross sections of the stars as compared with the total cross section of all the atoms of the interstellar gas.

The conditions within the interstellar gas might then be roughly compared to those in the positive column of a discharge tube with the stars behaving like floating electrodes. It seems justifiable, therefore, to use Ohm's law for weak fields.

The second assumption as to the existence of a steady state seems to be entirely reasonable.

The third assumption concerning the symmetry conditions is not unreasonable for the present purpose in view of the very small order of magnitude of the numerical results that are obtained. Of course, near a star, the stellar magnetic field is of importance and the electrons will not move in such a way as to allow the currents to be spherically symmetric. The magnetic field probably increases the density of electrons and ions near the star as well as the effective diameter of the star. For these reasons it is doubtful whether the effects of magnetic fields can be neglected without further consideration. However, it is not desirable at present to introduce the complications that would result from considering such effects. The magnetic field drops off rapidly with the distance away from the star, so by choosing r properly in the above equations the flow of current to the star can without excessive error be considered spherically symmetric.

To justify the fourth assumption for the present case, the following considerations must be made. The force acting on an electron in the neighborhood of a charged star is not caused exclusively by the presence of an electric field, but, rather, it is the resultant of the effects of gravity and radiation pressure in addition to a possible electric field. As soon as one introduces gravitational forces, however, it becomes necessary to consider diffusion. In the atmosphere surrounding a neutral star there is a balance between the rate at which particles are drawn toward the surface of the star by gravity, and the rate at which particles diffuse away from the surface because of the density gradient in the atmosphere. Included in this balance is the effect of radiation pressure. Now, if an electric charge is placed on the star, there will be an additional accelerating force applied to the charged particles surrounding the star, in particular to the electrons. This will alter the previous balance. The assumption made here is that the additional current produced by the electric field is related to the field in the manner described by Ohm's law. That this assumption represents only an approximation to the actual facts is evident, but it appears to be a closer approximation than would be obtained by introducing the effect of one of the above-mentioned factors, say gravity, without also considering the others.

Numerical Calculations

By substituting the following numerical values into Eq. (6)

$$e=4.8\times10^{-10}\text{ e.s.u.};\quad s=10^{-8}\text{ cm};\quad m=9\times10^{-28}\text{ g}$$

$$k=1.37\times10^{-16}\text{ erg/deg.};\quad T=10{,}000^\circ\text{K}$$

there results

$$V=-[1/(1.2\times10^{16}\beta)](A/a). \qquad (7)$$

To determine the value of A for the case of a star which only intercepts cosmic rays we shall assume that the particles move with a constant velocity c, the velocity of light, in random directions. Then the charge intercepted by the star per cm^2 per second, if all the cosmic-ray particles carry a charge of the same magnitude and sign is given by $Nce/4$, where N is the density of cosmic-ray particles and e is the magnitude of the charge on each. The total charge intercepted per second by a star of radius a is then given by

$$4\pi A=4\pi a^2Nce/4,\quad\text{and}\quad A=\tfrac{1}{4}Ncea^2.$$

If[2] $N=10^{-10}$ per cm^3, $c=3\times10^{10}$ cm per second and $e=4.8\times10^{-10}$ e.s.u., this becomes

$$A=3.6\times10^{-10}a^2\text{ e.s.u. sec.}^{-1}.$$

With this value of A and with $\beta=\frac{1}{2}$, it is found from Eq. (7) that a star of radius equal to that of the sun which is intercepting cosmic rays all of one sign will acquire an approximate potential of

$$V=1.3\times10^{-12}\text{ volt.}$$

Although the potential produced on each star as just calculated is very small it might be that

the galaxy would acquire a large potential as a result of the accumulative effect of such a charge on each star in the galaxy. The potential V on a spherical star of radius a is equivalent to a charge Va. So, if the galaxy is taken to be a sphere of radius R, and G is the total number of stars in the galaxy, the resultant potential of the galaxy, with respect to infinity, caused by an average charge Va on the stars is

$$V_G = GVa/R.$$

Then giving to V and a the same magnitudes as above and to G and R the values[7]

$$G = 1.45 \times 10^9; \quad R = 1.56 \times 10^{22} \text{ cm},$$

the potential produced on the galaxy as a result of stellar interception of cosmic rays is approximately

$$V_G = 8.4 \times 10^{-15} \text{ volt}.$$

To estimate A in the case of a star emitting cosmic rays, take as an upper limit to the energy so radiated the total energy of spectral radiation. Let the latter be represented by $4\pi S$ ergs per second. Then if each cosmic-ray particle represents energy of the amount W electron volts and carries a charge e e.s.u., the total amount of charge radiated per second is given by

$$4\pi A = 4\pi Se/(1.59 \times 10^{-12})W,$$

where 1.59×10^{-12} is the number of ergs in one ev. Then, if $e = 4.8 \times 10^{-10}$ e.s.u. and $W = 10^{10}$ ev,

$$A = 3.02 \times 10^{-8} S.$$

An average of several stars gives[8] $S = 5 \times 10^{34}$ ergs per second, so

$$A = 1.51 \times 10^{27} \text{ e.s.u. sec.}^{-1}.$$

By substituting into Eq. (7) it is seen that a star of radius equal to that of the sun emitting cosmic rays as charged particles all of one sign at the rate indicated will acquire a potential less than

$$V = 1086 \text{ volts}.$$

It is to be noted that in using Ohm's law for the conduction of electric currents through a gas the condition is imposed that the energy which an ion, or in this case an electron, acquires in traveling a distance equal to the length of a mean free path in the electric field must be small compared to the average energy of agitation of the atoms in the gas. The translation temperature of electrons in interstellar space corresponds to an energy of approximately one electron volt. This indicates that the energy acquired by an electron in traveling through a mean free path one terminus of which is at the surface of a star at a potential of 1000 volts will, if the electron's path is in the direction of a radius vector, be greater than the average energy of agitation of electrons by a factor of approximately 10^3. The case of a star that emits cosmic-ray energy of the amount indicated here as an upper limit cannot, therefore, be treated under the assumption of Ohm's law. This type of analysis will apply only to stars emitting cosmic rays at such a rate that the constant A has a value less than 10^{24} e.s.u. per second, resulting in a potential less than one volt. It should be remembered that it is not likely that a star will emit as much energy in the form of cosmic rays as it does in the form of heat.

Stellar Ion Sheaths

The analogy between a star immersed in the interstellar gas and a floating electrode in the positive column of a discharge tube can be used to determine what sort of ion sheath will be produced around a star as a result of its interception of the electrons and positive ions which make up a large part of the interstellar gas. This analogy was used in order to determine whether or not stellar ion sheaths would be of such dimensions as to influence the calculations already made.

Consider a single star and suppose it to be spherical in shape. It is placed in a gas made up of electrons and positive ions in equal number that move in random directions with a Maxwellian distribution of velocities the average of which corresponds to a translation temperature $T = 10{,}000°\text{K}$. Because of the higher mobility of the electrons the star will intercept in unit time more electrons than positive ions and hence become negatively charged. Positive ions will then be attracted to the star and a positive ion sheath will be formed in such a way that the

[8] Physics staff, University of Pittsburgh, *Atomic Physics* (Wiley, 1933), p. 291.

negative charge on the star will be just neutralized by the positive charge caused by all the ions in the sheath. In the final state the number of high energy electrons which are able to reach the star in unit time will be equal to the rate at which positive ions fall into the star as a result of its negative charge. To calculate the dimensions of the ion sheath use was made of the "orbital" and "space charge" equations developed by Langmuir and his co-workers in their theoretical study of spherical probes. These equations relate the total current flow of electrons and positive ions to the potential of the probe and the radius of its ion sheath.

If i_+ and i_- represent the total currents flowing to the star caused by positive ions and electrons, respectively, and I_+ and I_- are the current densities of these particles as intercepted by the star, the orbital equations are written[9]

$$i_+ = 4\pi r^2 I_+ [1-(1-b^2)\exp[-b^2 f/(1-b^2)]], \quad (8)$$

$$i_- = -4\pi r^2 I_- b^2 e^{-f}, \quad (9)$$

where, for the present case, a is the radius of the star, r is the radius of the ion sheath, b is the ratio a/r and $f=eV/kT$ which becomes $1.16V$, or approximately, $f=V$. The space charge equation is written[10]

$$i_+' = (4/9)(2e/M)^{\frac{1}{2}}\nu/\alpha^2, \quad (10)$$

where

$$\nu = V^{\frac{1}{2}}[1-0.0247(T/V)] \backsimeq V^{\frac{1}{2}},$$
$$\alpha = (\log b) - 0.38(\log b)^2 + \cdots,$$

and M is the positive ion mass which in this case is taken to be the mass of the hydrogen atom. Since the star corresponds to a floating electrode the net current flow to it is zero. On substituting Eqs. (8) and (9) into

$$i_+ + i_- = 0,$$

there results

$$1/b^2 - [(1-b^2)/b^2]\exp[-b^2 V/(1-b^2)] = 40e^{-V}, \quad (11)$$

where the ratio I_-/I_+, which is equal to the square root of the ratio of the masses of the hydrogen atom and the electron, is replaced by its approximate value 40.

[9] H. M. Mott-Smith and I. Langmuir, Phys. Rev. 28, 727 (1926).

[10] I. Langmuir and K. B. Blodgett, Phys. Rev. 24, 49 (1924).

Now substitute Eqs. (9) and (10) into

$$i_+' + i_- = 0$$

and obtain after solving for α and substituting numerical values

$$\alpha = 5.63 \times 10^{-7} V^{3/4} e^{V/2}. \quad (12)$$

It is found that Eqs. (11) and (12) will be solved simultaneously only for a value of b greater than 0.999. This indicates that if an ion sheath exists as a result of the interstellar gas, it is a very thin sheath.

Summary and Conclusions

The purpose of the investigation is to consider from a theoretical standpoint what electric fields would be set up if cosmic rays originated on stars and (or) traversed interstellar space as charged particles predominantly of one sign. Four assumptions are made regarding the conditions under which electric currents traverse interstellar space. In these assumptions account is taken of the astrophysical evidence for the existence of a finite density of matter in the space between the stars. The justification for these assumptions is discussed in some detail.

An equation is derived in which the electric field produced at a point in the neighborhood of a single star that intercepts, or emits, cosmic rays as charged particles all of one sign, is related to the rate at which the rays are intercepted, or emitted, by the star, the distance of the point from the center of the star, and the conductivity in interstellar space. It is shown that charge density and current density in interstellar space vanish.

Numerical calculations are made to determine what electric potential will result on a typical star as it intercepts, or emits, cosmic rays as charged particles under the conditions defined by the original assumptions. For the case of interception, the potential is found to be extremely small—of the order of 10^{-12} volt. The potential that would result on the galaxy from the combined average effect of all the stars in the galaxy is even smaller. In the case of emission, an upper limit to the rate at which energy might be emitted in the form of cosmic rays is taken to be the rate of spectral radiation. It turns out

that the assumptions made will not be valid for a star which emits cosmic-ray particles, all carrying the same charge, at this rate. The range of applicability is limited, in the case of a typical star, to rates of emission which are less than this indicated upper limit by a factor of at least 10^{-3}. This still might reasonably be termed an upper limit, and will result in producing a potential on the star of the order of one volt.

The calculations indicate, then, that the cosmic-ray particles will produce very small potentials on the stars. Since these potentials represent the only electric fields set up by the rays, one is led to the general conclusion that, as far as regards the electric fields that would be produced, theoretically it is possible for cosmic rays to originate, and to exist in interstellar space, as charged particles predominantly of one sign.

Acknowledgments

It is with pleasure that the author acknowledges the continual assistance of Professor Carl Eckart under whose supervision this research was carried out. Appreciation also is expressed to Professor A. H. Compton who suggested the problem, and to Professors Otto Struve and S. Chandrasekhar of the Yerkes Observatory and Doctor W. F. G. Swann of the Franklin Institute for helpful suggestions.

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